stant is as important as the ratio, ϵ_s/ϵ_f, and must be as small as possible to obtain complete circular polarization. With the increase of the dielectric constants (ϵ_s/ϵ_f = constant), the reflection at the receiving end surface (boundary between the plate No. 3 and the free space) increases and causes further multiple reflections and interactions between the sending and receiving end surfaces. It is noted also that the output axial ratio of polarizer No. 1 is extremely good, considering the depolarization which would be obtained for a circular polarized wave passing through a single plane interface. The above tells us that in this case the multiple reflections at the boundaries compensate each other as a whole so as to transmit a complete circular polarized wave over a wide frequency band.

The percentages of power reflected are plotted in Figs. 9 and 10. It is seen that better than 95 per cent transmission is obtained in all cases, and that the combinations yielding poorer axial ratios also exhibit the greater reflection.

Conclusions

A broad-band circular polarizer consisting of three anisotropic dielectric plates has been rigorously analyzed under the assumption that each plate has constant (different) values of refractive index along the two principal axes. As a result, the transmitted wave has been formulated in terms of the incident wave including interface reflections. The frequency characteristics of the axial ratio of polarization and the power transmission ratio have been numerically shown. From the above analysis it can be concluded that a combination of this type is very promising as a broad-band circular polarizer. The axial ratio is less than 1.075 and the reflected power less than 5 per cent over more than a 2:1 band of frequencies for each of the anisotropic dielectric plate combinations chosen as examples in this paper.

Acknowledgment

The assistance of Mrs. F. Phillips in the computation is gratefully acknowledged.

Large Signal Analysis of a Parametric Harmonic Generator*

KENNETH M. JOHNSON†

Summary—**Large signal analysis of a harmonic generator using a semiconductor diode reveals a larger possible efficiency than a similar small signal analysis. As higher harmonic numbers are reached, large signal analysis becomes increasingly more important in predicting the maximum conversion efficiency. It is shown that there exists an optimum value for the diode bias voltage and an optimum coupling of the load and generator to the diode, and that the diode operating voltage should almost drive the diode into conduction. An expression is derived for the maximum conversion efficiency for any harmonic, and it is shown that the conversion loss increases with increasing harmonic number, approximately 2.9 db per *n* for large harmonic numbers in a typical case.**

Introduction

Harmonic generators, unlike parametric amplifiers, are generally large signal devices; that is, the input signal traverses the entire dynamic range of the diode. For this reason, a large signal analysis must be used in the determination of the maximum conversion efficiency, especially at high harmonic numbers. Large signal analysis takes into account all the higher order contributions of the diode nonlinearity to the efficiency, and, as a result, a greater value for the maximum efficiency is obtained than is revealed by small signal analysis. The large signal analysis is reduced to the small signal equivalent[1] when higher order terms are not considered. This paper describes the large signal analysis of a semiconductor diode operating as a frequency converter and discusses the conditions required for maximum conversion efficiency.

Semiconductor Diode Harmonic Production

The Manley-Rowe[2] power relations indicate that a nonlinear reactance in a circuit can give rise to frequency conversion without losses. A back-biased semiconductor diode behaves as a nonlinear reactance in a circuit and, as a result, is useful for frequency conversion.[3] In this section we shall consider a diode as a harmonic generator to see how much energy conversion may be expected.

* Received by the PGMTT, April 6, 1960; revised manuscript received, June 28, 1960.
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[1] D. B. Leeson and S. Weinreb, "Frequency multiplication with nonlinear capacitance—a circuit anslysis," Proc. IRE, vol. 47, pp. 2076–2084; December, 1959.

[2] J. M. Manley and H. E. Rowe, "Some general properties of nonlinear elements—part I, general energy relations," Proc. IRE, vol. 44, pp. 904–913; July, 1956.

[3] D. Leenov and A. Uhlir, "Generation harmonics and subharmonics at microwave frequencies with *p-n* junction diodes," Proc. IRE, vol. 47, pp. 1724–1729; October, 1959.

The equivalent circuit for a semiconductor diode is shown in Fig. 1. The nonlinear capacitance of the diode $C_D(V)$ is given by[4]

$$C_D(V) = C_0'(1 - V/V_c)^{-m}, \tag{1}$$

where C_0' is the zero bias capacitance, V_c is the contact potential, V is the bias voltage on the diode, and m is equal to $\frac{1}{3}$ for a graded junction and $\frac{1}{2}$ for an abrupt junction. A typical value for m is 0.46. The diode's nonlinear resistance $G(V)$, which arises because the diode conducts at forward bias voltages, is given as

$$G(V) = \frac{qI_s}{kT} \exp \frac{qV}{kT}, \tag{2}$$

in which q is the charge on an electron, V is the bias voltage, and I_s is the reverse saturation current.

At reasonably low frequencies C_s in Fig. 1 may be neglected; a typical value for C_s might be 0.1 $\mu\mu$f. At negative bias voltages, $G(V)$ may be neglected in comparison with r_s, the spreading resistance.

In Fig. 2 we have the general capacitance vs bias voltage variation. For negative bias voltage, $G(V)$ is approximately zero and becomes significant at some forward voltage. Neglecting $G(V)$ and C_s, the equivalent circuit may be reduced to Fig. 3, where G_D is the parallel equivalent of r_s and therefore is frequency dependent. At reasonably high frequencies and for high diode Q, G_D is related to r_s and diode Q by

$$Q_D = \frac{\omega C}{G_D} = \frac{1}{\omega C r_s}. \tag{3}$$

Let V_B be the sum of the self bias and the applied dc voltage. If we define $V_0 = V_c - V_B$ and $K = C_0' V_c^m$, then (1) becomes

$$C_0 = KV_0^{-m} \tag{4}$$

in which C_0 is the operating point capacitance.

The diode, when operating as a harmonic generator, has only two sinusoidal voltages across it; all other frequencies are shorted out. One voltage, $V_1 \cos \omega_1 t$, appears at the fundamental frequency, ω_1, and the other voltage, $V_n \cos n\omega_1 t$, appears at $n\omega_1$, the nth harmonic. It will be shown later that when the diode operates at low efficiencies and at high harmonic numbers, V_n may be neglected in comparison with V_1.

If we assume a fixed bias on the diode and let ϕ_n be the phase difference which exists between V_1 and V_n, then the total voltage which appears on the diode is given as

$$V = V_B + V_1 \cos \omega_1 t + V_n \cos (n\omega_1 t + \phi_n). \tag{5}$$

We may now calculate the charge stored on the semiconductor junction due to these voltages. This charge is $q = \int C(V) dV$, or

$$q = -\frac{C_0 V_0}{1-m}\left[1 + \frac{V_1}{V_0}\cos \omega_1 t + \frac{V_n}{V_0}\cos (n\omega_1 t + \phi_n)\right]^{1-m} + q_0. \tag{6}$$

The charge may be represented in a Fourier series as

$$q = \frac{c_0}{2} + \sum_{n=1}^{\infty} (c_n \cos n\omega_1 t + d_n \sin n\omega_1 t). \tag{7}$$

This series may also be expressed in terms of complex exponentials as

$$q = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_n e^{in\omega_1 t} \tag{8}$$

The following relations exist between α_n, c_n, and d_n:

$$|\alpha_n| = \sqrt{c_n^2 + d_n^2}. \tag{9}$$

We may rewrite the charge using a phase as

$$q = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos (n\omega_1 t - \Psi_n), \tag{10}$$

in which

$$\Psi_n = \tan^{-1} \frac{d_n}{c_n} \quad \text{and} \quad a_n = |\alpha_n|. \tag{11}$$

The reason for utilizing this form will be obvious later.

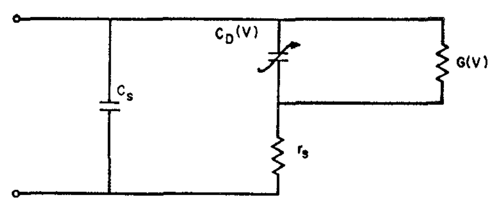

Fig. 1—Diode equivalent circuit.

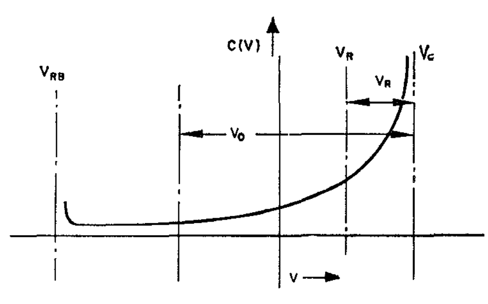

Fig. 2—Diode capacitance vs bias voltage.

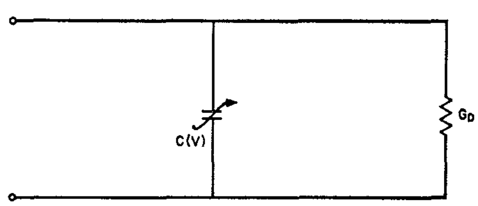

Fig. 3—Reduced diode equivalent circuit.

[4] A. Uhlir, "The potential of semiconductor diodes in high frequency communication," Proc. IRE, vol. 46, pp. 1099–1115; June, 1958.

For arbitrary values of V_1, V_n and ϕ, the Fourier integrals for d_n, c_n, and d_n can only be evaluated numerically or by series technique. However, since V_n is a direct result of V_1 and the driving source, the phase associated with V_n and the magnitude of V_n may be related to V_1, d_n and c_n. This will help fix the value of V_n and ϕ_n for maximum efficiency. A relatively simple technique for numerical integration of the Fourier integrals will be indicated in Appendix I. A series solution for these integrals is also possible. When V_n can be neglected, then $a_n = c_n$ and $d_n = 0$. The equation for c_n becomes

$$c_n = a_n = -\frac{C_0 V_0}{(1-m)\pi}\int_{-\pi}^{\pi}\left[1+\frac{V_1\cos\omega_1 t}{V_0}\right]^{1-m} \cdot \cos n\omega_1 t d(\omega_1 t). \tag{12}$$

This integral may be evaluated exactly by series technique, and for $V_1/V_0 = 1$ it may be integrated directly. By use of the binomial expansion we may obtain the following series solution to the integral. Let $1 - m = \gamma$, then

$$c_n = a_n = \frac{C_0 V_0(-1)^n(n-\gamma-1)!}{2^{n-1}n!(-\gamma)!}\left(\frac{V_1}{V_0}\right)^{n-1} \cdot\left[1+\sum_{k=1}^{\infty}\frac{n!(n-\gamma-1+2k)!}{k!(n+k)!(n-\gamma-1)!}\left(\frac{V_1}{2V_0}\right)^{2k}\right]. \tag{13}$$

This series converges very slowly for V_1/V_0 close to unity. However, for $V_1/V_0 < 0.5$ it may be evaluated quite easily. In fact, the higher order terms in k may be neglected. Neglecting these higher order terms gives essentially the small signal approximation.

A better evaluation of the charge can be made by representing it in a hypergeometric series. Sensiper and Weglein[5] have shown that the charge in (7) may be expressed in terms of a rapidly convergent hypergeometric series regardless of the value of V_1/V_0. For $\gamma = \frac{1}{2}$, the hypergeometric series reduces to complete elliptical functions which are available in tables.[6] The hypergeometric series which is rapidly convergent for $V_1/V_0 < 0.5$, is

$$q = \frac{-2C_0V_0}{\gamma}(1+t^2)^{-\gamma}t^n \cdot\frac{\Gamma(n-\gamma)}{\Gamma(-\gamma)\Gamma(n+1)}\,{}_2F_1(-\gamma, n-\gamma, n+1, t^2), \tag{14}$$

in which ${}_2F_1$ represents the hypergeometric series, Γ stands for the gamma function and t is defined from

$$\frac{V_1}{V_0} = \frac{2t}{1+t^2}. \tag{15}$$

[5] S. Sensiper and R. D. Weglein, "Capacitance and charge coefficients for parametric diode devices," *J. Appl. Phys.* (to be published).

[6] P. F. Byrd and M. D. Freidman, "Handbook of Elliptic Integrals for Engineers and Physicists," Springer-Verlag, Berlin, pp. 192–193; 1954.

Although for this hypergeometric series there are no tables available, it converges so quickly that it can easily be evaluated in a few terms.

General Power Relations

In this section we shall consider the general power relations for a harmonic generator to show the necessary circuit and the general equation for efficiency. In later sections we shall see how we may optimize the efficiency.

Fig. 4 shows the equivalent circuit for a harmonic generator.[7] The input circuit is resonant at ω_1 and coupled through a nonlinear capacitor to an output circuit resonant at $n\omega_1$, some multiple of ω_1. The input circuit consists of a resonant tank circuit T_1, resonant at ω_1 with a conductance G_1. Power is supplied by a current generator I_g, with generator conductance G_g across it. The output circuit consists of a tank circuit T_2, resonant at $n\omega_1$, with a loss conductance G_2 and a load conductance G_L across it. These circuits are coupled through the nonlinear capacitor C_D with a loss conductance G_D across it.

We shall assume that T_1 and T_2 are ideal filters. That is, T_1 presents a short circuit to current flowing at $n\omega_1$ and T_2 presents a short circuit to current flowing at ω_1. We may then represent the input circuit by Fig. 5, and the output circuit by Fig. 6, in which G_p is a positive conductance leading to energy transfer to the multiplied frequency and i_n is a current source at the nth harmonic due to the energy from G_p. We shall now determine G_p and i_n and apply them to the determination of the efficiency.

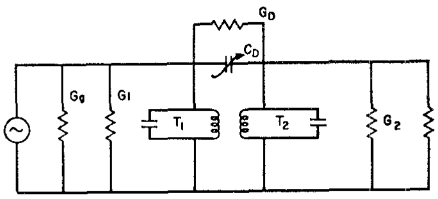

Fig. 4—Harmonic generator equivalent circuit.

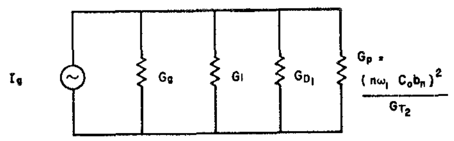

Fig. 5—Signal circuit equivalent circuit.

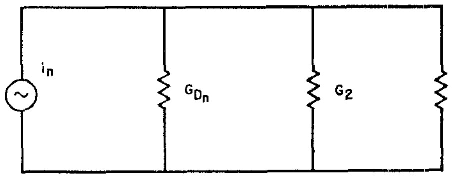

Fig. 6—Output circuit equivalent circuit.

[7] K. K. N. Chang, "Harmonic generation with nonlinear reactances," *RCA Rev.*, vol. 19, pp. 455–464; September, 1958.

The only frequencies which can exist are at ω_1 and $n\omega_1$,[8] so the charge on the nonlinear capacitor at $n\omega_1$ is simply given as

$$q_n = a_n \cos (n\omega_1 t - \Psi_n). \quad (16)$$

If we define $\phi_n = -(\Psi_n + 90°)$, then the current flow out of the capacitor at the right is

$$i_n = -\frac{dq_n}{dt} = n\omega_1 a_n \sin (n\omega_1 t - \Psi_n)$$

$$= n\omega_1 a_n \cos (n\omega_1 t + \phi_n). \quad (17)$$

This current supplies power to the output circuit conductances, G_L, G_{Dn} and G_2. We may calculate the power supplied by the diode at the multiplied frequency as

$$P_D = \frac{i_n^2 \text{ eff}}{G_{T_2}} = \frac{n^2\omega_1^2 a_n^2}{2G_{T_2}}, \quad (18)$$

where

$$G_{T_2} = G_L + G_{D_n} + G_2. \quad (19)$$

This power is supplied by V_1 at the fundamental frequency and is transferred through the positive conductance G_p.

For convenience let us define b_n such that

$$a_n = C_0 V_1 b_n. \quad (20)$$

We may now calculate the positive conductance G_p from (18) and (20),

$$G_P = \frac{n\omega_1 C_0 b_n^2}{G_{T_2}}. \quad (21)$$

If we define I_n as the magnitude of the current i_n flowing out of the diode and define V_n as the magnitude of the output voltage at the nth harmonic, then $I_n = V_n G_{T_2}$. The power taken off at the load is calculated as $V_n^2 G_L/2 = P_{\text{out}}$. The current generator I_g must supply all the input circuit conductances, G_g, G_1, G_{D_1} and G_p. The power available from the current generator is

$$P_{\text{av}} = \frac{I_g^2}{8G_g} = \frac{V_1^2(G_{T_1} + G_p)^2}{8G_g}, \quad (22)$$

where

$$G_{T_1} = G_1 + G_{D_1} + G_g. \quad (23)$$

The efficiency may now be calculated as

$$\eta = \frac{P_{\text{out}}}{P_{\text{av}}} 100 = \frac{2G_L G_g}{G_{T_1} G_{T_2}} \cdot \frac{100}{\left[1 + \frac{(n\omega_1 C_0 b_n)^2}{2G_{T_1}G_{T_2}} + \frac{G_{T_1}G_{T_2}}{2(n\omega_1 C_0 b_n)^2}\right]}. \quad (24)$$

Matching Conditions for Optimum Efficiency

Let us investigate the equation for efficiency, to determine the conditions for optimum efficiency, by using the following definitions for coefficients and Q's:[9]

$$Q_g = \frac{\omega_1 C_0}{G_g}$$

$= $ external Q of signal circuit loaded only by G_g,

$$Q_L = \frac{n\omega_1 C_0}{G_L}$$

$= $ external Q of output circuit loaded only by G_L,

$$Q_{D_1} = \frac{\omega_1 C_0}{G_{D_1}}$$

$= $ external Q of signal circuit loaded only by G_{D_1},

$$Q_{D_n} = \frac{n\omega_1 C_0}{G_{D_n}}$$

$= $ external Q of output circuit loaded only by G_{D_n},

$$\mu = \frac{Q_{D_1}}{Q_g},$$

$$\nu = \frac{Q_{D_n}}{Q_L}.$$

For maximum efficiency, we must minimize the circuit losses G_1 and G_2 so that $G_1 \ll G_{D_1}$, $G_1 \ll G_g$, $G_2 \ll G_{D_n}$, and $G_2 \ll G_L$. In most microwave or UHF circuits these conditions are met. That is, G_1 and G_2 are usually negligible. By assuming this to be the case and realizing that in a physical diode $Q_{D_n} = Q_{D_1}/n$, we may write the efficiency as

$$\eta = \frac{2\mu\nu}{(1+\mu)(1+\nu)} \cdot \frac{100}{\left[\frac{1 + Q_{D_1}^2 b_n^2}{2(1+\mu)(1+\nu)} + \frac{(1+\mu)(1+\nu)}{2Q_{D_1}^2 b_n^2}\right]}. \quad (25)$$

[8] J. Hilibrand and W. R. Beam, "Semiconductor diodes in parameter oscillators," *RCA Rev.*, vol. 20, pp. 229–253; June, 1959.

[9] R. C. Knechtli and R. D. Weglein, "Low noise parametric amplifier," PROC. IRE, vol. 47, p. 584; April, 1959.

By differentiating (25) with respect to μ and setting it equal to zero, we obtain the optimum value of μ. Doing this, we find that the generator conductance should be matched to the other signal circuit conductances, *i.e.*, $G_g = G_{D_1} + G_1 + G_p$. This is rather obvious, since for maximum power transfer the generator is always matched to the input circuit.

For the output circuit we would expect ν or G_L/G_{D_n} to be at a maximum in order to minimize losses in G_{D_n}. However, since $G_L + G_{D_n} + G_2$ enters into the value of G_p, which was to be made large in order to minimize losses due to G_1 and G_D, then G_{T_2} must be small. Thus ν has an optimum value which may be determined in the same manner as for μ. This turns out to be a function of μ, and when $\mu = \mu_{\text{opt}}$ and $\nu = \nu_{\text{opt}}$, it may be written

$$\mu = \mu_{\text{opt}} = \nu = \nu_{\text{opt}} = \sqrt{1 + Q_{D_1}{}^2 b_n{}^2} \tag{26}$$

and

$$\eta_{\text{opt}} = \frac{\mu_{\text{opt}} - 1}{\mu_{\text{opt}} + 1} = \frac{\sqrt{1 + Q_{D_1}{}^2 b_n{}^2} - 1}{\sqrt{1 + Q_{D_1}{}^2 b_n{}^2} + 1} \,. \tag{27}$$

From this expression it may be seen that for maximum efficiency $Q_{D_1} b_n$ should be as large as possible. In Fig. 7 we have plotted η vs Q_{D_1} for various values of b_n.

In Fig. 8 we have plotted η vs ν assuming $\mu = \sqrt{Q_{D_1}{}^2 b_n{}^2 + 1}$, to show the importance of ν. Since ν is always somewhat greater than unity, the load should be overcoupled to the diode.

We shall now consider the diode itself to see how factor $Q_{D_1} b_n$ can be maximized in order to obtain greater efficiencies.

Optimizing the Diode Parameters

The maximum value of the factor $Q_{D_1} b_n$ for any particular circuit is dependent only on the diode and its bias voltage. To maximize the diode Q alone, a diode should be chosen with a low series resistance and low capacitance. Since the diode Q is defined at the operating point capacity C_0, and not at the zero bias capacity, a diode should be chosen with as large a reverse breakdown voltage V_{RB} as possible, so that C_0 will be small. Thus, the maximum value V_0 may take is approximately $V_{RB}/2$.

From (9), (10), and (17) it is readily seen that to maximize b_n, V_1 (or the sum of V_1 and $V_n \cos \phi$ at $\omega_1 t = \pi$) must approach V_0. That is, we want to drive the capacity to infinity. However, as the capacitance goes to infinity, so do the forward conduction losses $G(V)$. Thus, the diode voltages may be limited[8] by some voltage V_R at which point the forward losses become so excessive as to reduce the diode Q. We may define V_R as the voltage where the diode Q is reduced to

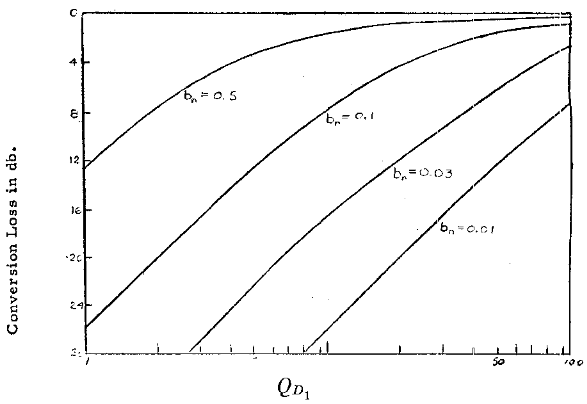

Fig. 7—Conversion loss vs diode Q for various values of the Fourier coefficient b_n.

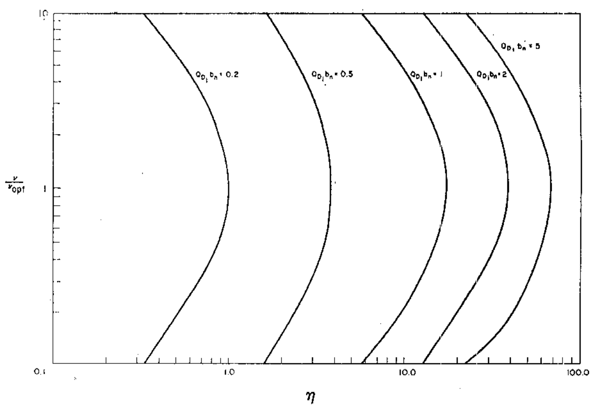

Fig. 8—Efficiency vs normalized load to diode conductance coupling.

half its original value due to the presence of $G(V)$. From Fig. 1 we can compute the total Q of the diode considering the effects of $G(V)$ and r_s:

$$\frac{1}{Q_{D_T}} = \frac{1}{Q_{D_1}} + \frac{1}{Q_{D_G}} \tag{28}$$

where

$$Q_{D_T} = \text{total diode } Q$$

$$Q_{D_1} = \frac{1}{\omega C(V) r_s} = \text{diode } Q \text{ due to } r_s \text{ only}$$

$$Q_{D_G} = \frac{\omega C(V)}{G(V)(1 + r_s G(V))} \,.$$

Measured from V_0, V_R is then defined at the voltage where $Q_{D_1} = Q_{D_G}$. Normally $r_s G(V) \ll 1$ so V_R may be defined under the condition that

$$C(V) = \omega^2 C^2(V) r_s, \tag{29}$$

or for $m = \frac{1}{2}$ when

$$\frac{kT\omega^2 C_0'^2 \phi r_s}{q i_s} = V_R \exp \frac{q(\phi - V_R)}{kT} \,. \tag{30}$$

The instantaneous sum of the applied voltage at $\omega_1 t = \pi$ may be written by inspection of (6) as $V_1 = (-1)^{(n-1)} V_n \cos \phi_n$. The maximum value of this in terms of V_R may be written

$$\left[\frac{V_1 + (-1)^{n-1} V_n \cos \phi}{V_0}\right]_{\max} = 1 - \frac{V_R}{V_0} = 1 - \frac{2V_R}{V_{RB}}. \quad (31)$$

For a maximum b_n, we want then a diode with a large $V_{RB}/2V_R$. The importance of this large value is shown in Fig. 9 where we have plotted $V_{RB}/2V_R$ as a function of b_n assuming $V_n = 0$. The assumption that $b_n = 0$ applies only to the case of low efficiencies as will be shown in the next section. When V_n is considered, the value of b_n becomes somewhat larger and the effect of $V_{RB}/2V_R$ becomes less pronounced. In Fig. 10 we have shown how b_n may vary as a function of V_1/V_n for the particular case that $V_1 + (-1)^{n-1} V_n \cos \phi_n = V_0$.

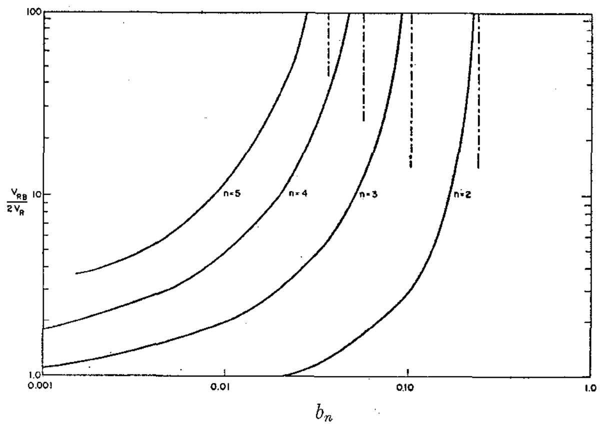

Fig. 9—Variation of Fourier coefficient with the diode parameter $V_{RB}/2V_R$.

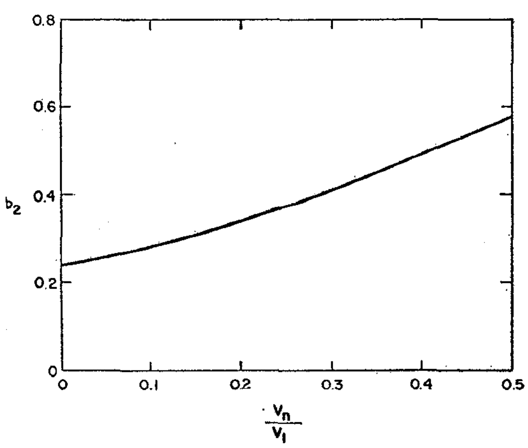

Fig. 10—Ratio of harmonic voltages vs normalized Fourier coefficient b_n.
$V_1 + (-1)^{n-1} V_n \cos \phi = V_0$

MAGNITUDE OF NTH HARMONIC VOLTAGE

We shall show from the general circuit equations how V_1 is related to V_n so that we may determine whether or not V_n may be omitted from (6).

Since we are ordinarily concerned with operation at maximum efficiency, we use the general circuit equations for optimum efficiency to relate V_1 to V_n. From (14) and the fact that $I_n = V_n G_{T_2}$ we may write

$$\frac{V_n}{V_1} = \frac{n\omega_1 C_0 b_n}{G_{T_2}}. \quad (32)$$

Now

$$G_{T_2} = G_{D_n}(1 + \gamma_{\text{opt}}) = G_{D_n}\left[1 + \sqrt{1 + Q_{D_1}{}^2 b_n{}^2}\right], \quad (33)$$

so that

$$\frac{V_n}{V_1} = \frac{Q_{D_1} b_n}{n\left[1 + \sqrt{1 + Q_{D_1}{}^2 b_n{}^2}\right]} = \frac{\sqrt{\eta}}{n}. \quad (34)$$

Therefore V_n/V_1 is always less than $1/n$ and, when operating with low efficiencies, $V_n \ll V_1$. Thus for $n > 10$ and efficiencies less than 1 per cent, $V_n/V_1 < 0.01$ and may be neglected without appreciably changing the determined value for efficiency. In the next section we shall show how the efficiency may be determined from the foregoing equations.

DETERMINATION OF EFFICIENCY

When V_n is negligible in the case of low efficiencies, we may use the value of b_n as determined from the hypergeometric series. Knowing, then, the diode Q we can solve for the efficiency from (27). If we consider the effect of V_n, then the equation we use for the "normalized" Fourier coefficient b_n as given from (7), (9) and (20) is given as

$$b_n = \sqrt{c_n'^2 + d_n'^2}, \quad (35)$$

where

$$c_n' = \frac{-V_0}{V_1 \pi(1 - m)} \int_{-\pi}^{\pi} \left[1 + \frac{V_1 \cos \omega_1 t}{V_0} + \frac{V_n}{V_0} \cos(n\omega_1 t + \phi_n)\right]^{1-m} \cos n\omega_1 t \cdot d\omega_1 t. \quad (36)$$

$$d_n' = \frac{-V_0}{V_1 \pi(1 - m)} \int_{-\pi}^{\pi} \left[1 + \frac{V_1 \cos \omega_1 t}{V_0} + \frac{V_n}{V_0} \cos(n\omega_1 t + \phi_n)\right]^{1-m} \sin n\omega_1 t \cdot d\omega_1 t. \quad (37)$$

Recall that

$$\phi_n = -\left(90° + \tan^{-1} \frac{d_n'}{c_n'}\right). \quad (38)$$

We may now solve for b_n if V_1 and V_n are known. Eq. (34) is another equation relating V_1 and V_n to b_n. This equation may be expressed in a more useful form as

$$b_n = \frac{2nV_n}{Q_{D_1}V_1} \frac{1}{1 - \frac{n^2V_n^2}{V_1^2}}. \quad (39)$$

To solve for b_n and η, it is necessary to try various values of V_1 and V_n until (34) and (35) are satisfied. Essentially, then, we must solve by trial and error (36), (37), (38), and (39) for any particular diode Q. Although this may seem quite tedious, it can be done. In the following section we shall show how important it is to consider V_n in a practical case. The appendixes provide the means for solution of the integrals. A series technique is useful if one has access to a computer.

Maximum Efficiency

In this section we shall consider a practical diode to show what the maximum efficiency would be as a function of harmonic number.

Let us consider a typical Hughes parametric amplifier diode, the HPA 2800 diode, with an operating point capacity of 1 $\mu\mu$f, a r_s of 4 ohms, and $V_{RB}/2V_R$ of 40. Instead of including G_1 and G_2 in the equation for efficiency, we shall assume for simplicity that the input and output filters have a total loss of 0.3 db. By numerical integration of the integral for the Fourier coefficient, we are able to plot conversion loss vs n for a fundamental frequency of 1 kmc; this is done in Fig. 11. The dotted portion of the curve shows the effect when V_n is neglected and only the hypergeometric series used. It may be seen that the conversion loss increases as would be expected and for large n the conversion loss increases approximately 2.9 db per n.

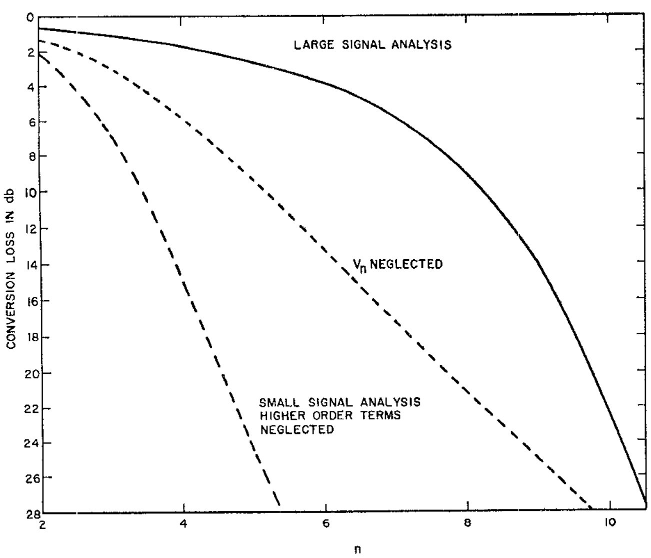

Fig. 11—Theoretical curves of conversion loss vs harmonic number for a typical Hughes diode. $C_0=1$ $\mu\mu$f; $r_s=4$ ohms; $V_{RB}/2V_R=40$, which corresponds for $n=2$ to $P_{in}\approx 13$ milliwatts; 0.3-db filter losses.

Experimental work that the author has done has indicated that it is difficult to obtain efficiencies much greater than those indicated by the equations which neglect V_n for harmonics greater than the second. This may have been due to some fault in the diode used which showed itself at higher microwave frequencies, or perhaps to a poor estimation of the Q of the diode used. However, comparison was made with the results of Chang[7] and these give a little better agreement with the theory. Using the values they give for diode Q and assuming that the parameter $V_{RB}/2V_R$ for their diode was equal to 10, the following calculations were made: The diode Q for $n=2$ was calculated to be about 5, for $n=3$ to be about 7.5. The Fourier coefficients b_2 and b_3 were calculated to be 0.250 and 0.053 respectively. This gave a maximum efficiency for doubling of 23 per cent and for tripling of 3.65 per cent. If the filter losses were about 0.2 db, then the expected efficiencies are 22 per cent and 3.47 per cent. This is to be compared with the experimental results of 22 per cent for doubling and 2 per cent for tripling. Thus, the accuracy is quite good for doubling and not quite as good for tripling.

Conclusion

Good efficiencies may be obtained if the harmonic generator is operated so that the signal drives the diode somewhat into conduction and uses as large a bias voltage as possible on the diode. A diode should be chosen which has a large Q, a large reverse breakdown voltage, and a small voltage difference between the point of infinite capacity and high forward conduction. If the coupling of the load to the diode is optimized and the generator is critically coupled to the signal circuit, optimum matching conditions are achieved. As operation is extended to higher and higher harmonic numbers, the nonlinear contribution to the efficiency becomes exceedingly important and the diode chosen must have a high reverse breakdown voltage and low voltage difference between infinite capacity and high forward conduction.

By following these principles, it is possible to obtain efficiencies at least as good as those indicated by the equations which neglect V_n. As better diodes are obtained, it is likely that it will be possible to achieve the theoretical maximum efficiency for all harmonics. Because of the good efficiencies which this analysis shows are possible for large harmonic numbers, it may be, in some cases, just as efficient to convert directly to a large harmonic number as to use several doublers or triplers in cascade. This will give a saving on the quantity of diode and filters required.

Appendix I

In this appendix we shall indicate a technique for numerical integration of (36) and (37). Since the graph of the function to be integrated is merely a cosine wave in $n\omega_1 t$, modulated by a uniformly varying function, it is only necessary for the most part to determine the magnitude of the amplitude of the cosine wave from $\omega_1 t = 0$ to $\omega_1 t = \pi$. At $\omega_1 t = \pi$ an additional correction may have to be made if $V_{RB}/2V_R$ is not ∞. The integral of the function will be proportional to the sum of these amplitudes, provided that the distance over which the half cosine waves occur is taken into consideration.

As a simple example, if we consider (12) for $n=2$ and $V_1/V_0 = 1$, we find we have three peaks occurring approximately at 0, $\pi/2$ and $3\pi/8$. This function is represented in Fig. 12. The respective magnitudes are 2, -1, and 0.19. The integral is then given by

$$b_2 = \frac{4}{\pi}\left[\frac{\sqrt{2}}{2} - 1.0 + \frac{0.19}{2}\right] = 0.252.$$

The actual value by direct integration is $b_2 = 0.240$; we see the accuracy is fairly good.

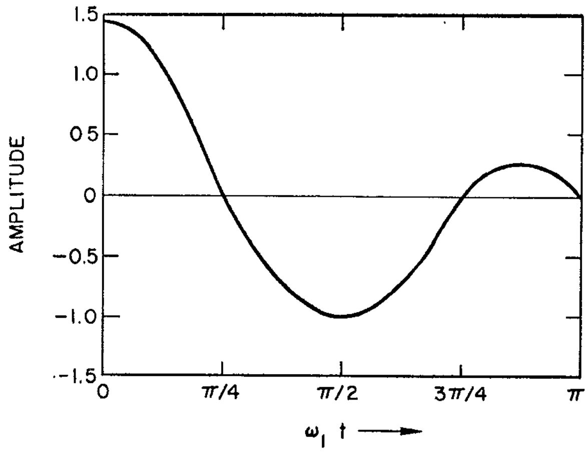

Fig. 12—Variation of second harmonic charge for computation of normalized Fourier coefficient b_n.

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A Waveguide Switch Employing the Offset Ring-Switch Junction*

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Summary—This paper describes an X-band microwave ring switch employing a junction of new design. The insertion loss is less than ¼ db over the design band of 9.0–9.6 kmc; this low loss was obtained through the use of a coupling junction which is designated the offset ring-switch junction. A physical description, the electrical characteristics of the switch, and a qualitative theoretical discussion of the junction are included.

Introduction

RAPID scanning of a sector at a constant rate with a narrow beam antenna, by movement of the complete antenna or by motion of a feed mechanism, imposes the problem of minimizing the dead-time between scans. Many scanning antennas employ multiple feeds or dishes which rotate at constant speed to limit the dead-time to a reasonable value without forcing the antenna or scanning mechanism to undergo rapid acceleration between scans. Several types of microwave switches have been employed for sequentially switching the input waveguide of such antennas to each of the several feeds or dishes. Waveguide ring switches are often employed for this purpose.

Peeler and Gabriel reported a waveguide ring switch[1] which is based on an annular rotary joint patented by Breetz.[2] This switch employs right-angle bends made of interleaving pins which project into the ring waveguide parallel to the E field, and couple energy into, and out of, the ring guide. This coupling junction requires the

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[2] L. D. Breetz, U. S. Patent No. 2,595,186, April 29, 1952. Also, L. D. Breetz, "A Waveguide Rotary Joint with Waveguide Feed," Naval Res. Lab., Washington, D. C., NRL Rept. 3795; January 18, 1951.